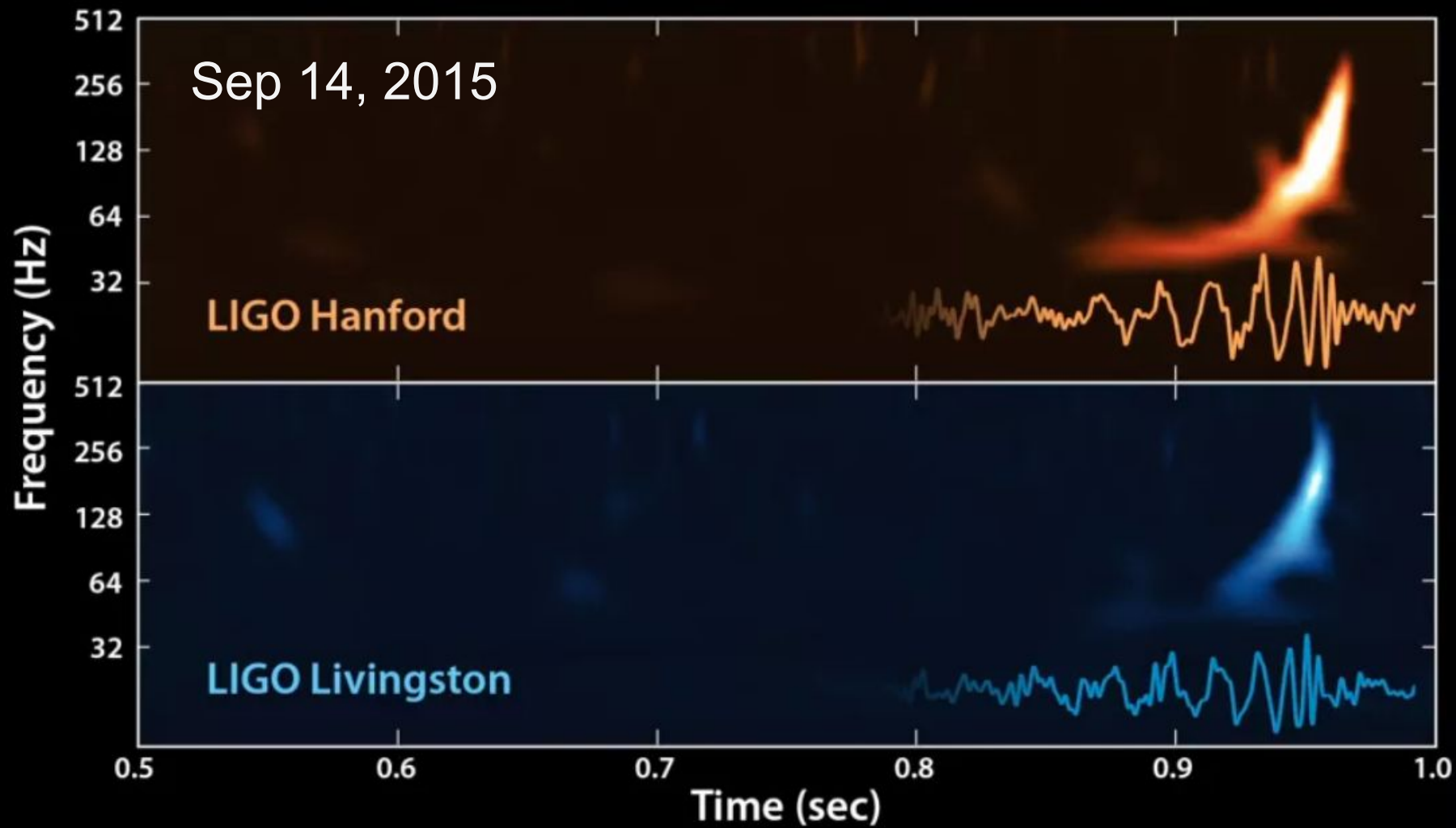


Counting Black Holes

The Cosmic Stellar Remnant Population and
Implications for LIGO

Elbert, Bullock, and Kaplinghat (2018)



We are pivoting from "GW *discovery* to GW *astronomy*"

Previous work:

Lamberts et al. (2016):

- GW 150914 likely formed in massive galaxy at $z \sim 1$
- Dwarf galaxy also possible

Belczynski et al. (2016):

- Massive black holes prefer low metallicity galaxies

Chatterjee et al. (2017):

- Massive black holes formed in globular clusters prefer massive, low metallicity clusters

What can we **observe** (about binary stellar mass black holes)?

Currently observable

- Rate
- BH Masses
- Spins

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- Merger redshifts

Difficult but possible

- Distribution of host galaxy properties (SFR, galaxy mass, metallicity, etc.)
- Host galaxies of individual mergers

What can we **predict** (about binary stellar mass black holes)?

Assuming stellar population synthesis + merger "efficiency" + merger timescale:

As a function of BH mass:

- Rate
- Distribution of host galaxy masses / metallicities / SFRs
- Distribution of BBH *formation* redshifts

What can we **infer** (about binary stellar mass black holes)?

Currently observable

- Rate
- BH Masses
- Spins
- Merger redshifts

Difficult but possible

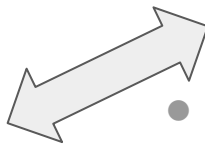
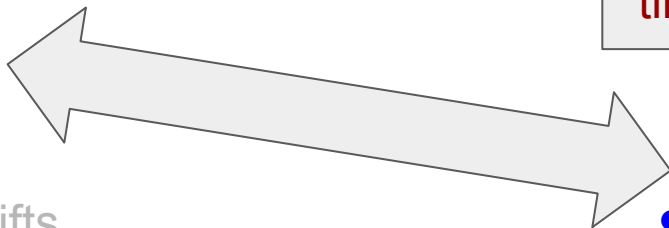
- Distribution of host galaxy properties (SFR, galaxy mass, metallicity, etc.)
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Stellar population synthesis +

merger "efficiency" + merger timescale:

As a function of BH mass:

- Rate
- Distribution of host galaxy masses / metallicities / SFRs
- Distribution of BBH formation redshifts



How do we go about inferring these things?

Assume all black holes are from stellar evolution

Jean

Estimate N (black holes) as a function of BH mass & galaxy properties

Yashvi

Parameterize your ignorance about (1) the fraction that merge
(2) the time it takes to merge

Kishalay

Compare predictions with observables to constrain parameters

What determines the black hole number density?

1. How massive must a star be to form a black hole?
2. How many progenitor stars exist in a galaxy?
3. What is the density of galaxies of a given total stellar mass?

Black hole masses will be denoted with m_{bh}

Stellar masses will be denoted with \mathcal{M}

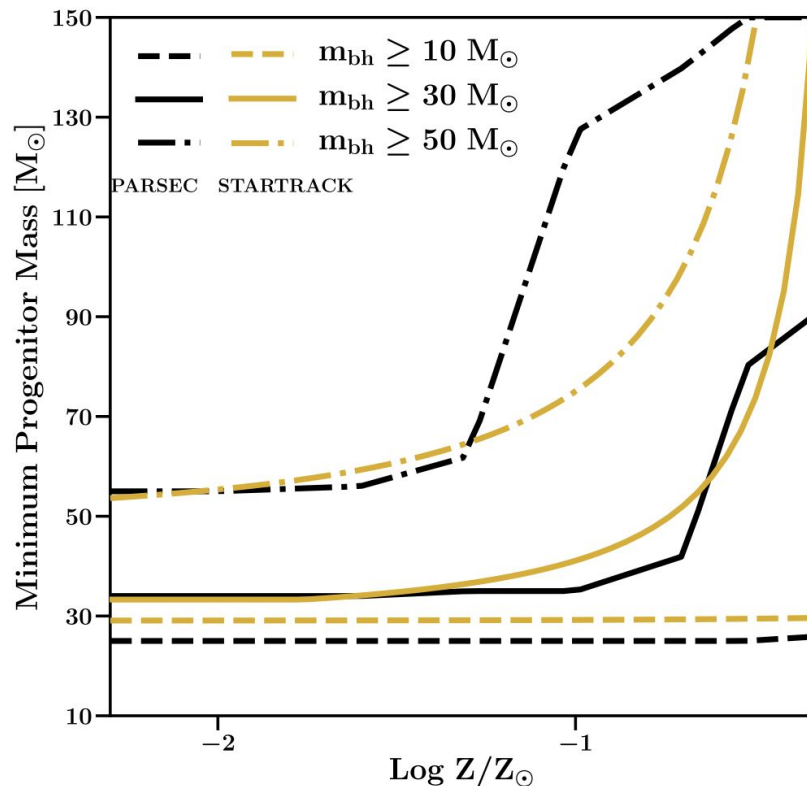
Total galaxy stellar masses will be denoted with M_{\star}

1. Minimum stellar mass to form a black hole

$$\mathcal{M}_{\min}(m_{\text{bh}}, Z)$$

Note: strong dependence on metallicity

Fiducial values use PARSEC



2. How many progenitor stars exist in a galaxy?

**Fraction of stars of a given
metallicity which could form black
holes $> m_{\text{bh}}$**

Fiducially $150 M_{\odot}$

$$\int_{\mathcal{M}_{\min}(m_{\text{bh}}, Z)}^{\mathcal{M}_{\text{u}}} \xi(\mathcal{M}') d\mathcal{M}'$$

Stellar IMF

Kroupa 2002

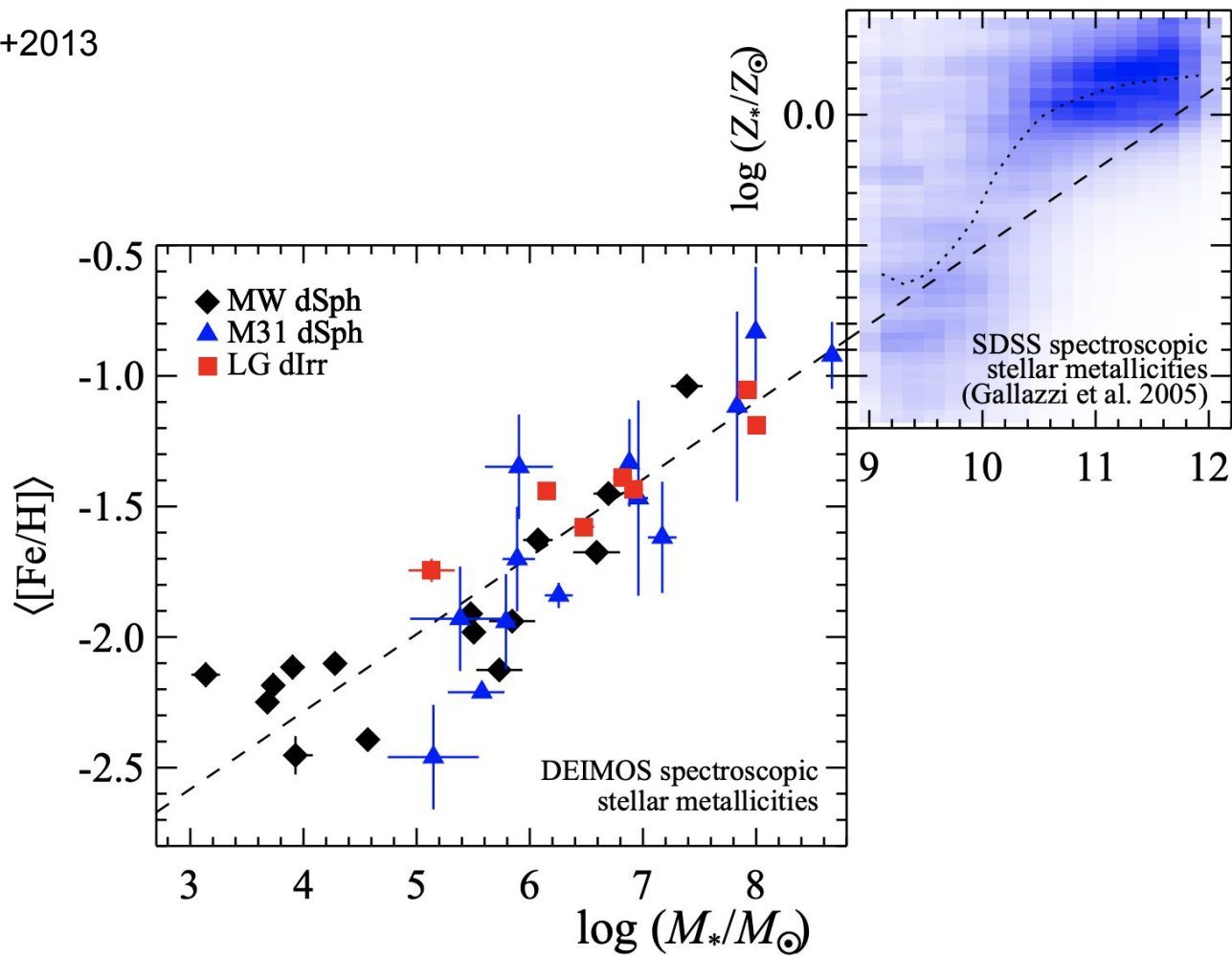
2. How many progenitor stars exist in a galaxy?

Fraction of stars with
metallicity Z in a galaxy of
mass M_*

$$\int \mathcal{P}(Z, M_*) \int_{\mathcal{M}_{\min}(m_{\text{bh}}, Z)}^{\mathcal{M}_{\text{u}}} \xi(\mathcal{M}') d\mathcal{M}' dZ$$

Metallicity distribution
function

Gallazzi+2005, Kirby+2013



2. How many progenitor stars exist in a galaxy?

Normalization

$$N_*(M_*) = \frac{M_*}{\int_{0.08 M_\odot}^{\mathcal{M}_l(M_*)} \mathcal{M}' \xi(\mathcal{M}') d\mathcal{M}'}$$

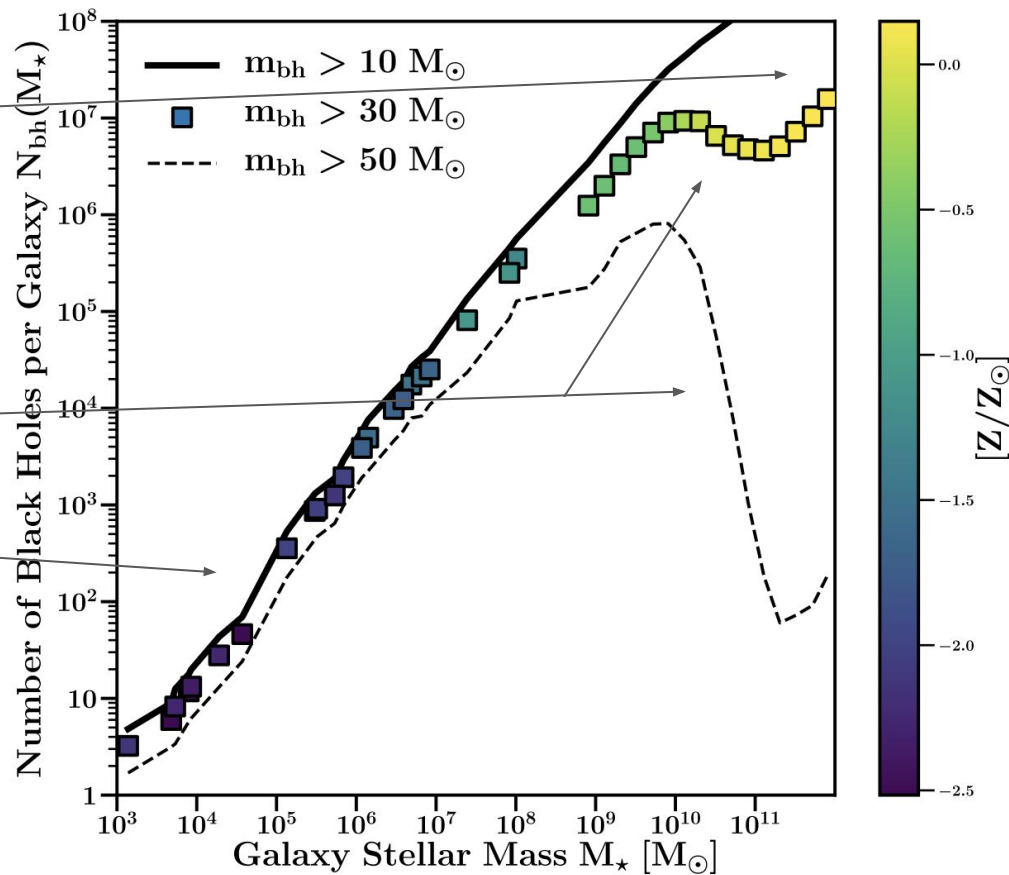
$$N_\star(M_\star) \int \mathcal{P}(Z, M_\star) \int_{\mathcal{M}_{\min}(m_{\text{bh}}, Z)}^{\mathcal{M}_u} \xi(\mathcal{M}') d\mathcal{M}' dZ$$

The upper bound is the mass for which MS lifetime =
average galaxy age
(Behroozi, Wechsler & Conroy 2013)

MDF flattens

**Metallicity
effects kick in**

**Linear
relation**



$$N_{\text{bh}}(>m_{\text{bh}}, M_{\star})$$

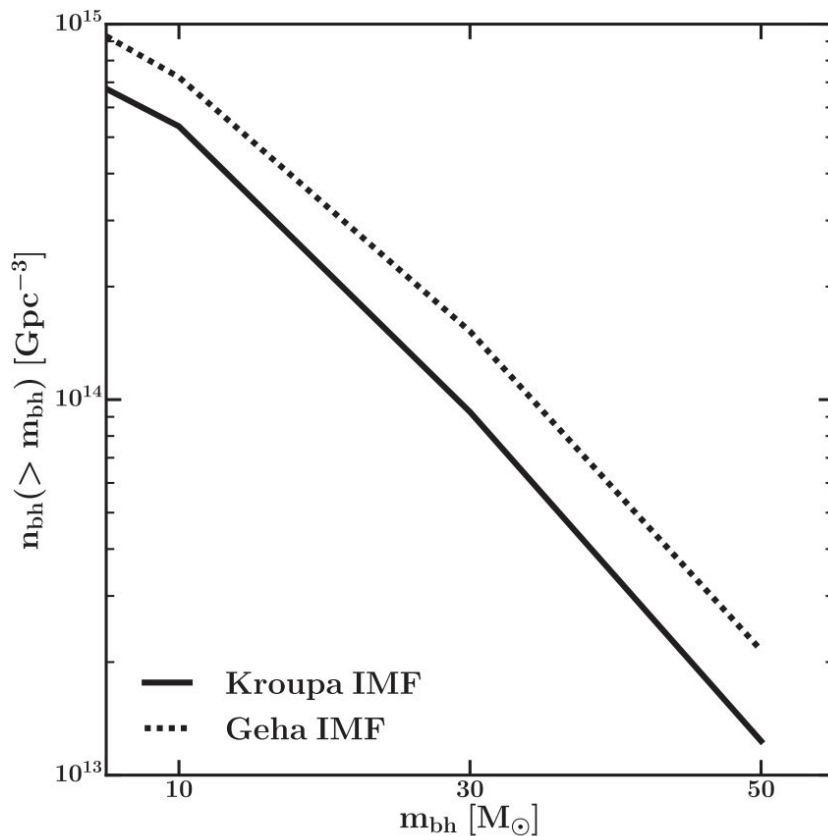
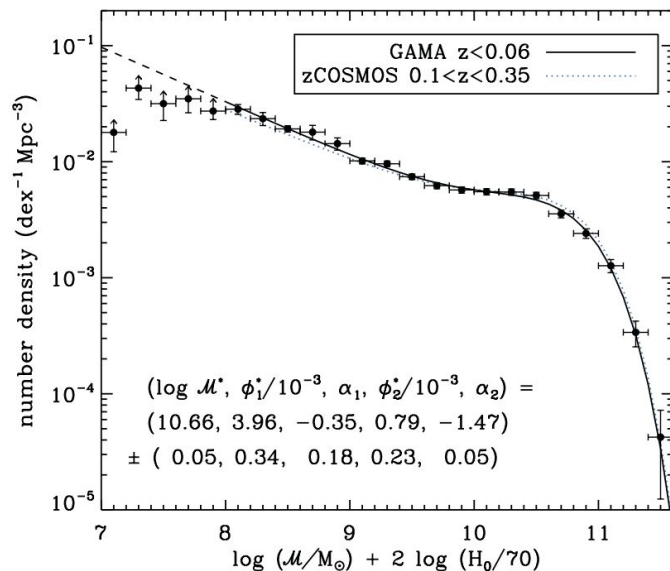
$$= N_{\star}(M_{\star}) \int \mathcal{P}(Z, M_{\star}) \int_{\mathcal{M}_{\min}(m_{\text{bh}}, Z)}^{\mathcal{M}_{\text{u}}} \xi(\mathcal{M}') d\mathcal{M}' dZ$$

3. How many galaxies are there?

$$n_{\text{bh}}(>m_{\text{bh}}) = \int_{M_{\text{min}}}^{\infty} \phi(M_{\star}) N_{\text{bh}}(>m_{\text{bh}}, M_{\star}) dM_{\star}$$

Galactic Stellar Mass Function

Baldry et al. 2012, $M_{\text{min}} = 10^3 M_{\odot}$

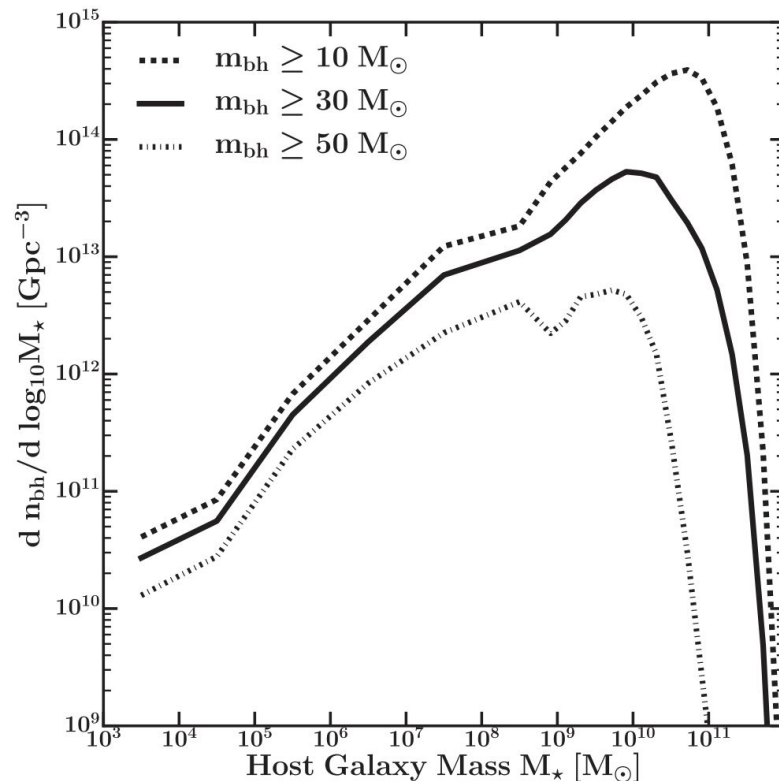


Lower mass black holes live in the most massive galaxies

High mass black holes live in dwarfs

Verification that this methodology is reasonable:

Applying the same equations to core collapse supernovae, they recover the observed number density within a factor of ~ 1.5



From BH number density to merger rate

Assumptions:

1. Mergers occur amongst binary pairs

- 'Binary black hole efficiency' parameter : ϵ

$$n_{\text{bh}}^{\text{pair}}(>m_{\text{bh}}) = \frac{1}{2} \epsilon n_{\text{bh}}(>m_{\text{bh}}).$$

- Both BHs in a pair above same threshold mass i.e. $m_1, m_2 > m_{\text{bh}}$

2. Mergers occur after birth of binary pair over time-scale τ

Parameters to constrain : ϵ and τ

Epsilon magic

$$\epsilon \equiv f_{b\star} \times f_{m_1/m_2} \times f_{\text{surv}} \times f_t$$

Massive star
binary fraction

~ **0.5**

*Kobulnicky & Fryer
(2007)*

Fraction of
massive binary
systems with
 $m_1/m_2 = 0.9$ is

~ **0.1**

Sana et al. (2012)

Fraction of binary
systems that
survive as BH
pair after stellar
evolution ~ **0.1**

Belczynski et al. 2016a

Fraction of
BBH available
to merge
before
present day

$f_t < \mathbf{1}$

Black hole merger rate density

- Formation rate density of BH pairs that can merge

$$\dot{n}_{\text{bh}}^{\text{pair}} = 0.5 \epsilon \dot{n}_{\text{bh}}$$

- Birth-rate density of BH
 - Assuming that it tracks the observed shape of global SFR density

$$\dot{n}_{\text{bh}}(>m_{\text{bh}}, t) = n_{\text{bh}}(>m_{\text{bh}}) \frac{\psi(t)}{\int_0^{t_0} \psi(t') dt'}$$

- Then for a distribution of merger times $P(\tau') = \delta(\tau' - \tau)$

$$\mathcal{R} = \frac{1}{2} \epsilon \dot{n}_{\text{bh}}(t_0 - \tau)$$

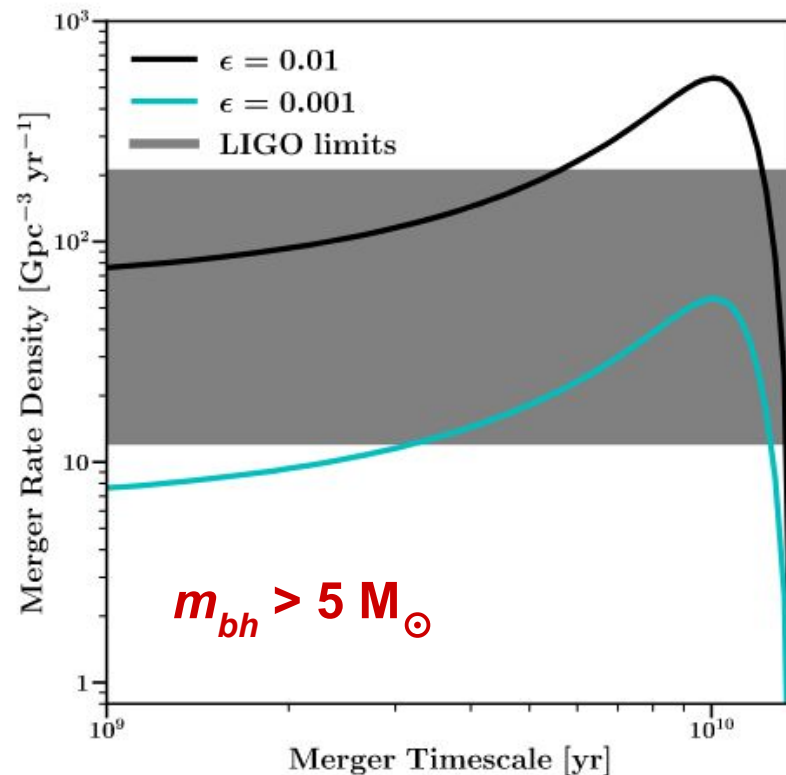
Predicted merger rate density

Gray region : $12 \leq R \leq 213$ (Abbott et al., 2017)

Shorter τ (< 2 Gyr) needs larger ϵ (1%)

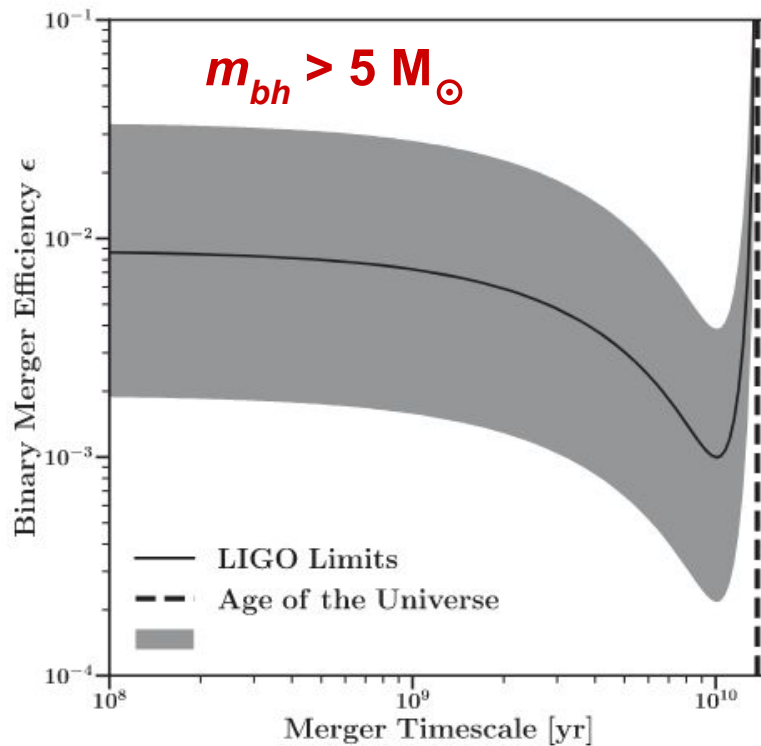
Longer τ (~ 10 Gyr) needs smaller ϵ (0.1%)

Similar agreement for $m_{bh} > 30 M_{\odot}$



Degeneracy between τ and ϵ

$\epsilon \sim 0.2 - 3 \%$
For $\tau < 2 \text{ Gyr}$



$\epsilon \sim 0.02 - 0.4 \%$
For $\tau \sim 10 \text{ Gyr}$

NS-NS & Massive BH ($>50 M_{\odot}$) merger rates

From Abbott et al. (2016b)

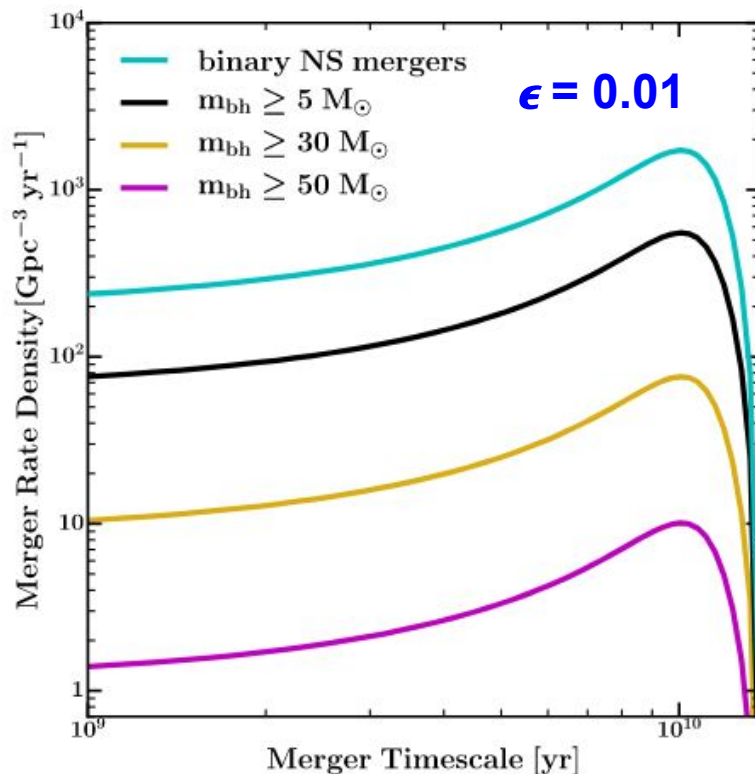
$$\mathcal{R}_{\text{NS}} < 12\,600 \text{ Gpc}^{-3} \text{ yr}^{-1}.$$

From Kim, Kalogera & Lorimer(2006) &
Enrico Petrillo, Dietz & Cavagli`a (2013)

$$\mathcal{R}_{\text{NS}} \simeq 10^2 - 10^3 \text{ Gpc}^{-3} \text{ yr}^{-1}$$

Predicted rate density for $m_{\text{bh}} > 50 M_{\odot}$

$$\mathcal{R}_{50} \gtrsim 1 (\epsilon/0.01) \text{ Gpc}^{-3} \text{ yr}^{-1}$$



Breaking the degeneracy between τ and ϵ

Host galaxy populations of a statistical sample of mergers can constrain the delay times and merger efficiencies:

- Short delay times = Young and smaller star forming galaxies
- Long delay times = Old and larger quenched galaxies

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$$\mathcal{R}_{\text{prompt}} = \frac{1}{2} \epsilon \dot{n}_{\text{co},0}.$$

Galaxy mass function

$$\dot{n}_{\text{ns},0} = \int_{M_{\text{min}}}^{\infty} \phi(M_{\star}) \int \frac{\dot{M}_{\star}(M_{\star}, Z_{\text{g}})}{\bar{\mathcal{M}}} \mathcal{P}(Z_{\text{g}}, M_{\star})$$

Initial mass function

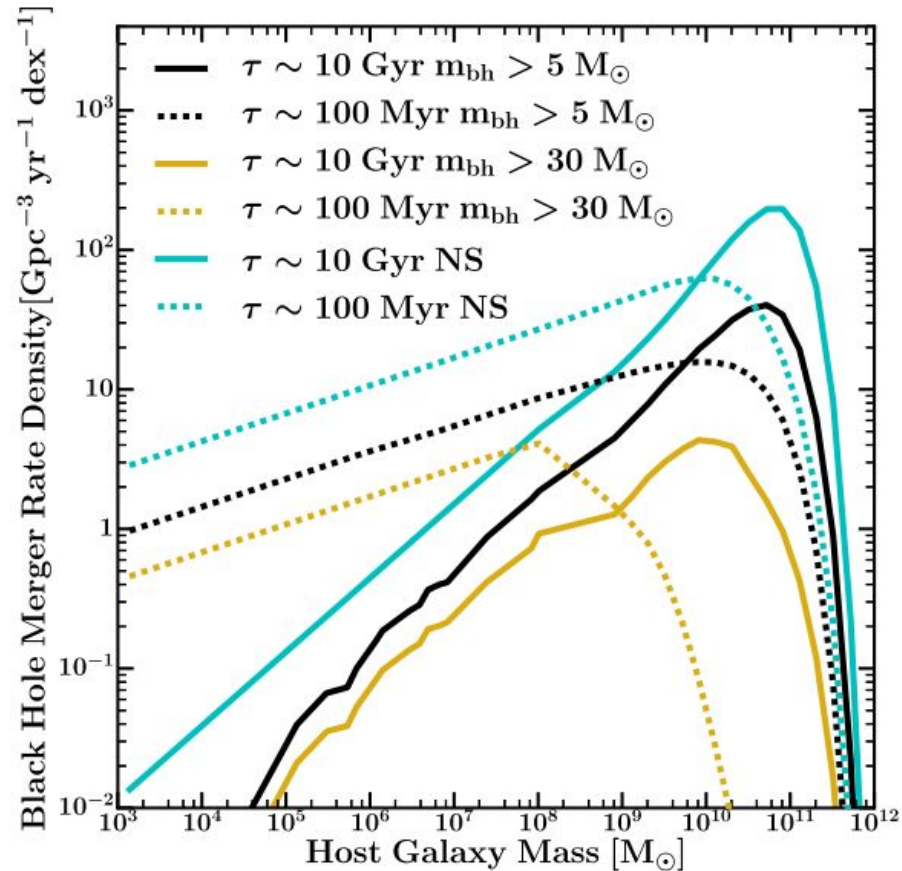
$$\int_{\mathcal{M}_{\text{min}}^{\text{ns}}}^{\mathcal{M}_{\text{max}}^{\text{ns}}} \xi(\mathcal{M}') d\mathcal{M}' dZ_{\text{g}} dM_{\star}.$$

Number density of star formation

Metallicity distribution

The diagram illustrates the components of the equation for the prompt rate. The top equation is $\mathcal{R}_{\text{prompt}} = \frac{1}{2} \epsilon \dot{n}_{\text{co},0}$. Below it, the equation for $\dot{n}_{\text{ns},0}$ is shown: $\dot{n}_{\text{ns},0} = \int_{M_{\text{min}}}^{\infty} \phi(M_{\star}) \int \frac{\dot{M}_{\star}(M_{\star}, Z_{\text{g}})}{\bar{\mathcal{M}}} \mathcal{P}(Z_{\text{g}}, M_{\star})$. An arrow points from 'Galaxy mass function' to $\phi(M_{\star})$. Another arrow points from 'Initial mass function' to the integral over \mathcal{M}' . A third arrow points from 'Number density of star formation' to $\dot{M}_{\star}(M_{\star}, Z_{\text{g}})$. A fourth arrow points from 'Metallicity distribution' to $\mathcal{P}(Z_{\text{g}}, M_{\star})$.

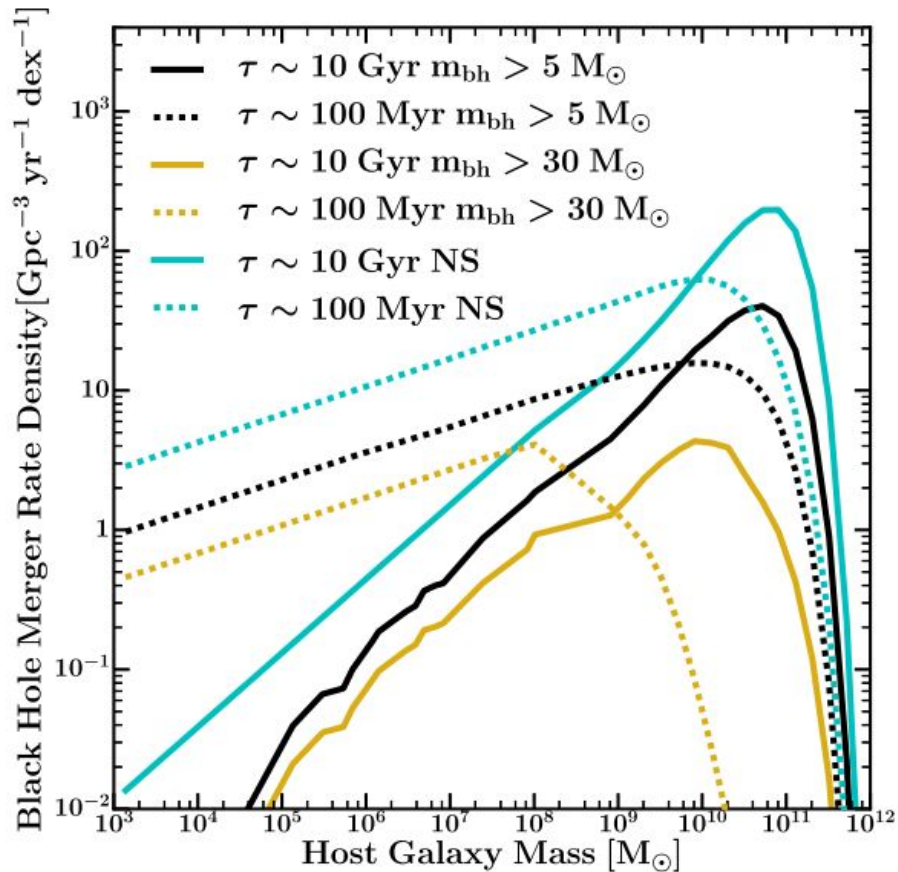
Merger rate as a function of host galaxy mass



Merger rate as a function of host galaxy mass

Shorter timescale mergers occur in lower mass galaxies on average (stars forming today)

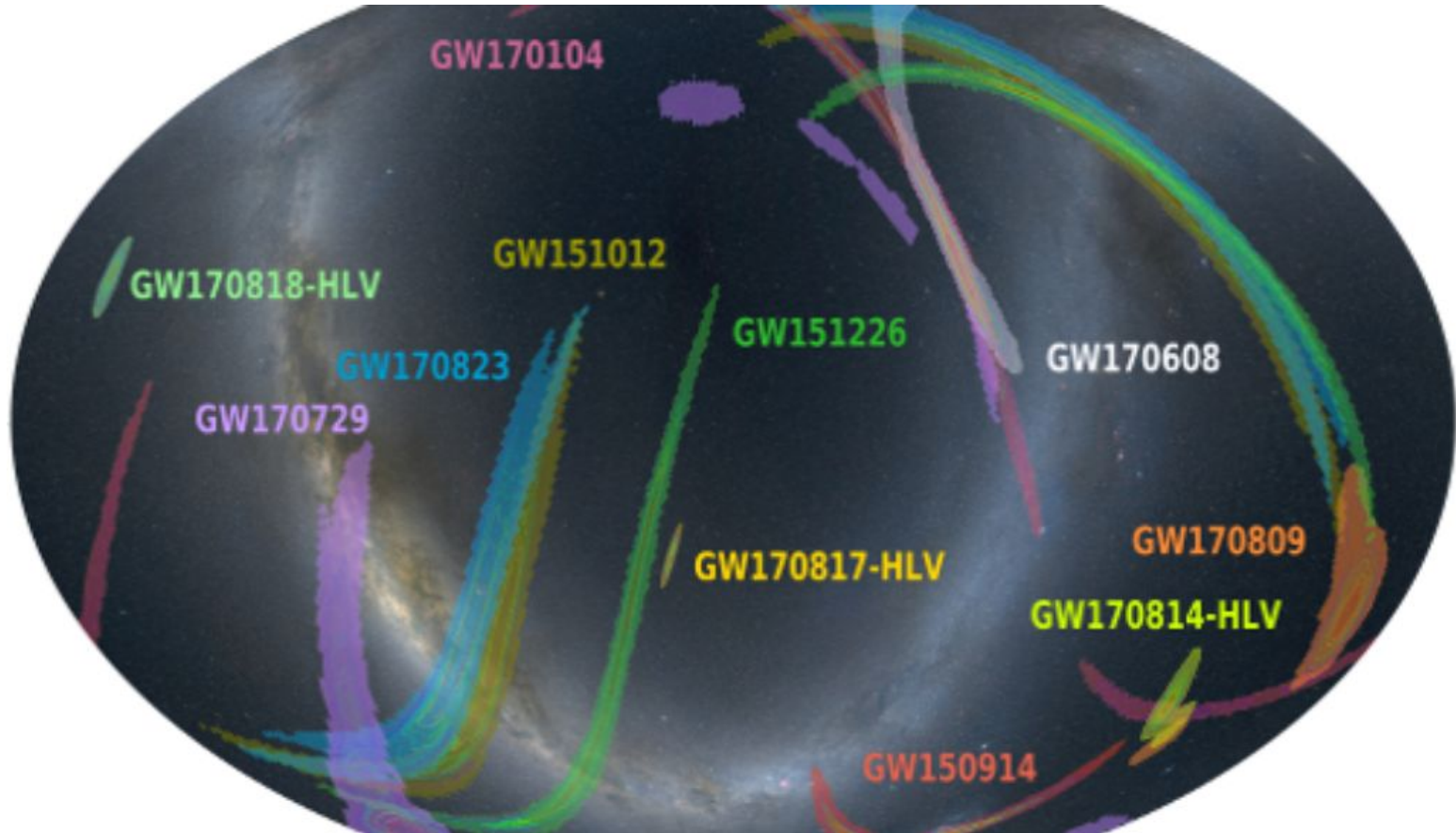
Strong effect for massive BHs since low metallicity only in low mass galaxies



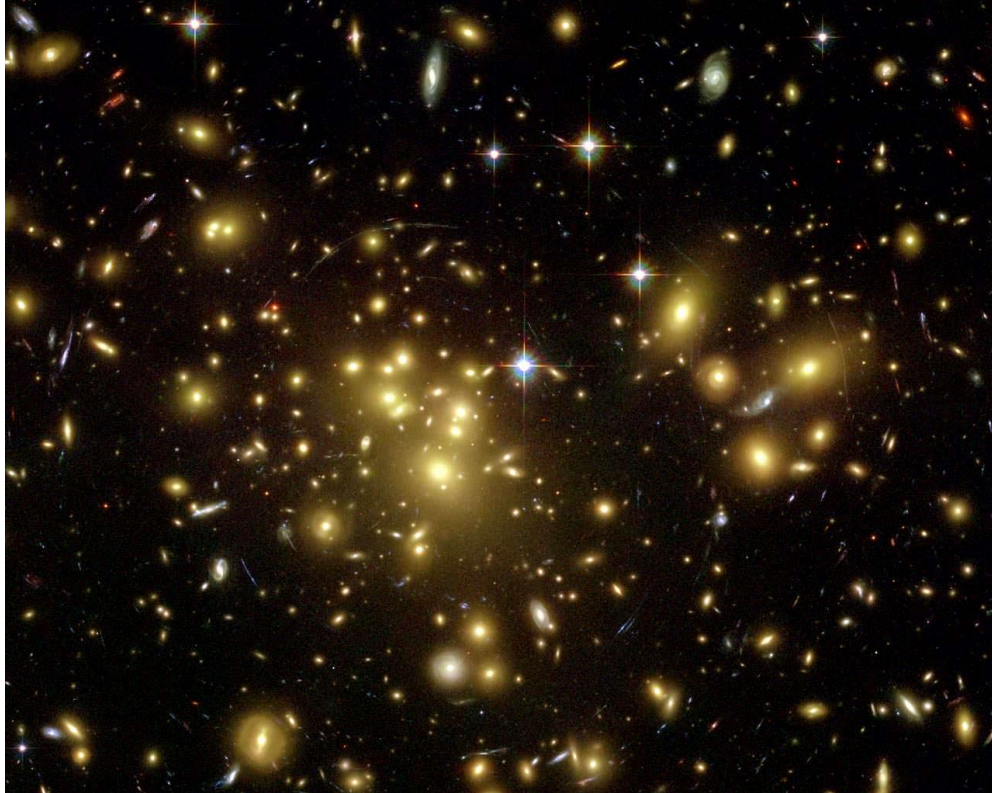
47% of NS-NS mergers in dwarf galaxies for 100 Myr; Only 6% in dwarfs if 10 Gyr

51% of BBH mergers in dwarf galaxies, only 9% in dwarfs is 10 Gyr

The testing challenge: EM counterparts / localizations



The testing challenge: EM counterparts / localizations



- Very massive galaxies preferentially hosted in giant clusters
- Even poorly localized association with clusters can test the fraction of BBH / BNS mergers in massive galaxies and constrain delay time and efficiencies.

What have we learned?

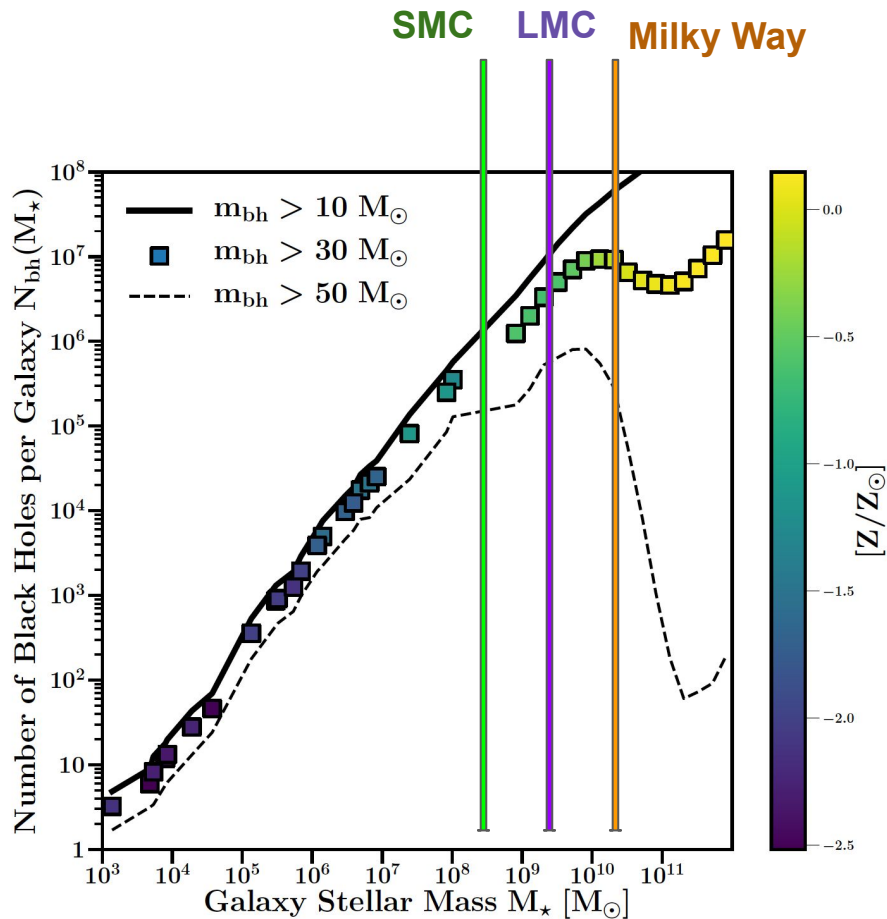
Estimate N (black holes) as a function of BH mass & galaxy properties

- $N_{\text{BH}} = 0.9 - 2 \times 10^{14} \text{ Gpc}^{-3}$
- For galaxies $< 10^{10} M_{\odot}$:
 - One $> 30 M_{\odot}$ BH
per $1000 M_{\odot}$ of stars
- Plenty of massive BHs to merge

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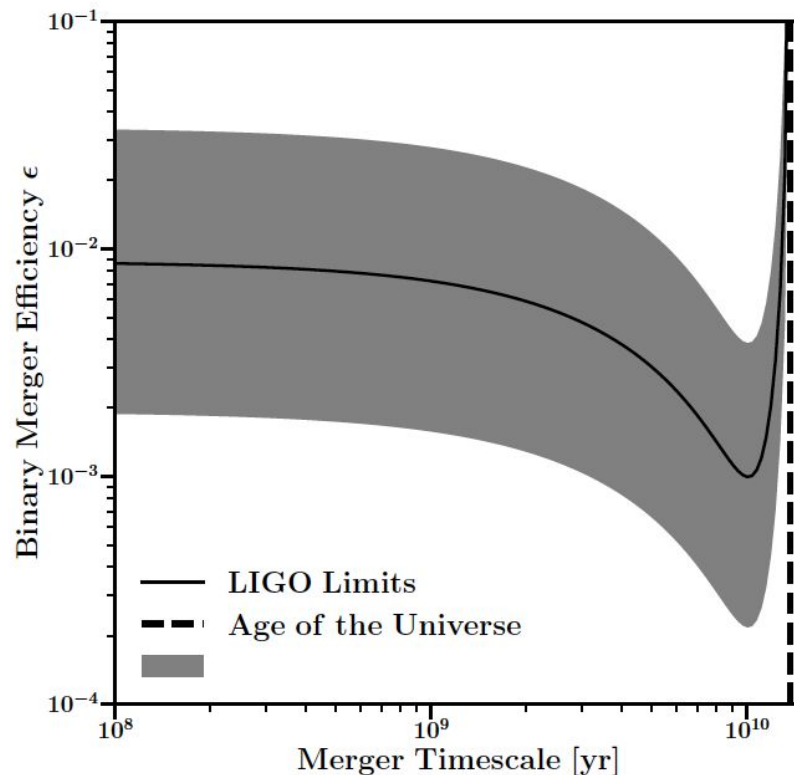
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 - Plenty of massive BHs to merge
- ... especially in dwarf galaxies!



What have we learned?

Parameterize your ignorance about (1) the fraction that merge
(2) the time it takes to merge

- Short merger timescales (< 5 Gyr) mean that $\sim 1\%$ of BH's are in binaries that merge
- Efficiency drops as merger timescales get longer, but spikes back up as it approaches a Hubble time



What have we learned?

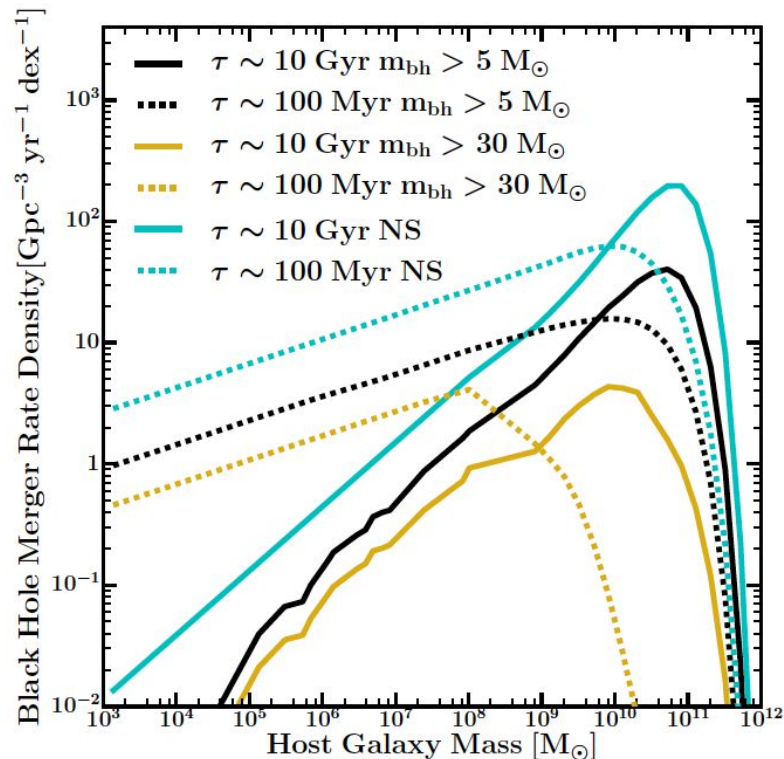
Compare predictions with observables to constrain parameters

One possible way to break this degeneracy is to identify the host galaxy mass distribution for observed merger events. Small galaxies today have ongoing star formation, while larger galaxies tend to be quenched (e.g. Mannucci et al., 2010). Thus, binary mergers that occur soon after formation will more likely be seen in small galaxies. Mergers over timescales comparable to the age of the universe, however, will more closely track the overall stellar mass distribution. Most stars are in massive galaxies today (Baldry et al., 2012; Bernardi et al., 2013). Thus mergers detected locally that have take a long time to occur will be biased to reside within large galaxies.

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What have we learned?

Assume all black holes are from stellar evolution

(2012; Lan et al., 2016). Scaling from the event-based rate derived for GW150914, we would therefore predict the rate for $50 M_{\odot}$ black holes binary mergers to be $\mathcal{R}_{50} = 8^{+27}_{-6} \text{ Gpc}^3 \text{ yr}^{-1}$. This

Corresponding to ~ 56 per Gpc per year for > 30 solar masses

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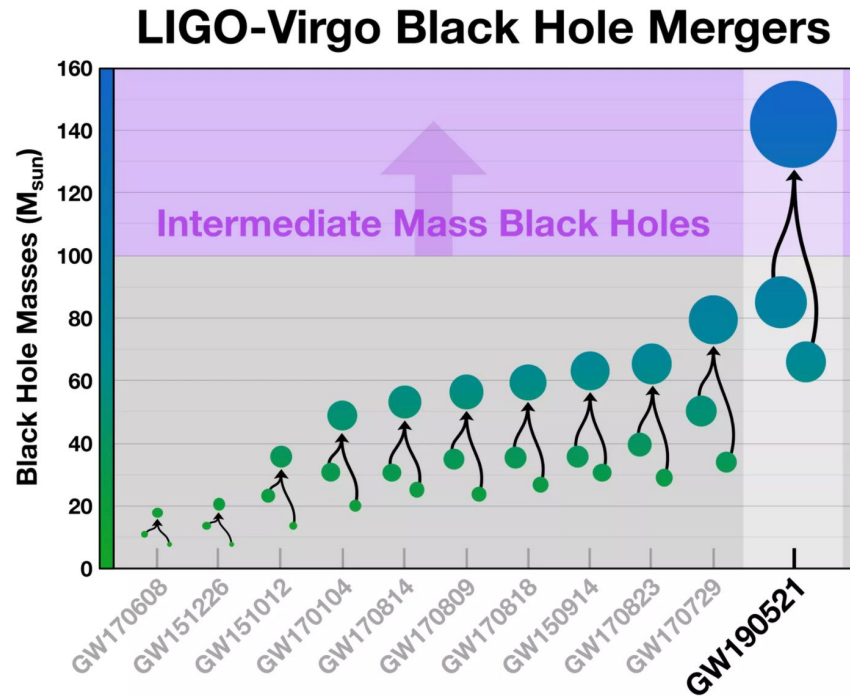
GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs

of marginal event candidates with an estimated false-alarm rate less than 1 per 30 days. From these results over the first two observing runs, which include approximately one gravitational-wave detection per 15 days of data searched, we infer merger rates at the 90% confidence intervals of $110 - 3840 \text{ Gpc}^{-3} \text{ y}^{-1}$ for binary neutron stars and $9.7 - 101 \text{ Gpc}^{-3} \text{ y}^{-1}$ for binary black holes assuming fixed population distributions and determine a neutron star–black hole merger rate 90% upper limit of $610 \text{ Gpc}^{-3} \text{ y}^{-1}$.

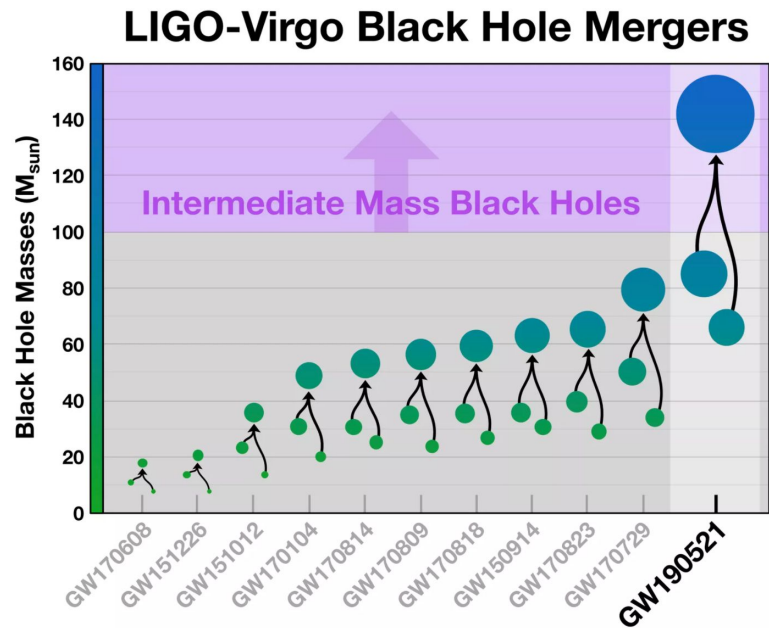
**Not bad, given O1 + O2 constraints.
However...**

Assume all black holes are from stellar evolution

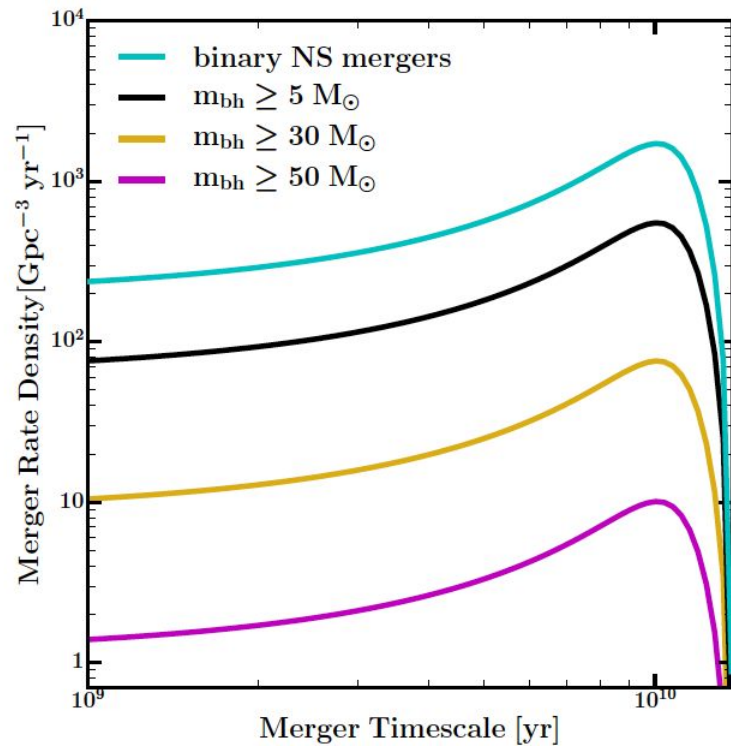
Though our approach is not well suited for ab initio calculations, it does provide fairly robust scalings because the uncertain/unknown parameters are reasonably constant for all compact objects in our calculations. For example, for any ϵ or τ , $50 M_{\odot}$ black holes should have merger rate densities that are a factor of 7 ± 1 smaller than merger rates of binary $30 M_{\odot}$ black holes (see Fig. 7). This range accounts for uncertainties in the faint end of the



Assume all black holes are from stellar evolution



rate of mergers similar to GW190521 is $0.13^{+0.30}_{-0.11} \text{ Gpc}^{-3} \text{ yr}^{-1}$



Rates aren't quite strong enough yet to challenge the stellar evolution assumption.

However, pair instability + other arguments (spins? asymmetry?) could be.

Questions?

